Amendments to Specifications:

Please replace the title of the application on page 1 with the following title:

~~SKILLS-BASED~~ CONTACT CENTER SCHEDULING USING INTEGER PROGRAMMING

Please replace the U.S. Patent Documents cited on page 1 with the following citations:

U.S.PATENT DOCUMENTS

5,216,593	6/1993	*	Dietrich et al.	345/467
5,911,134	6/1999		Castonguay et al.;	364/401
6,044,355	3/2000		Crockett et al.	705/8
6,192,346	2/2001		Green	364/401
6,278,978	8/2001		Andre et al.	705/7
6,622,134	9/2003	*	Sorkin	706/20
2004/0054564	3/2004	*	Fonseca et al.	705/007
6,856,680	2/2005	*	Mengshoel et al.	379/265.06
2006/0112049	5/2006	*	Mehrotra et al.	706/046
7,249,047	7/2007	*	Arguello et al.	705/9

* cited by examiner

Please replace the title of the application on page 3 with the following title:

~~SKILLS-BASED~~ CONTACT CENTER SCHEDULING USING INTEGER
PROGRAMMING

Please replace paragraph 1 on page 3 with the following amended paragraph:

The invention pertains to the field of computer systems and computer
implemented methods. More specifically, the invention pertains to methods <u>and</u>
<u>computer systems</u> for scheduling agents with multiple skills optimally at contact centers
to satisfy time varying agent staffing requirements for multiple contact types.

Please replace paragraph 2 on page 6 with the following amended paragraph:

U.S. Pat. No. 6,044,355 issued on March 28, 2000 to Crockett et al. describes a
simulation method for developing weekly tour schedules in a contact center environment
involving multiple agent groups with a plurality of skill sets, and a plurality of contact
types requiring different agent skills. The method uses a scheduler and an Automatic
Call Distributor (ACD) simulator. However, no working details or description of the
scheduler, and how it develops a schedule is included in the patent document. It was
disclosed that the method uses a "scheduler program" ~~such as TotalView™ available~~
~~from IEX Corporation~~ (U.S. Pat. No. 6,044,355, col. 6, lines 18-36). No evidence to
suggest that the schedules obtained using the scheduler program ~~TotalView™~~
<u>considered</u> will satisfy the necessary and sufficient conditions for optimality in a
workforce scheduling problem is provided or known.

Please replace paragraph 2 on page 7 with the following amended paragraph:

A primary object of the present invention is to overcome these limitations of the
prior art agent scheduling methods. The present invention uses Mixed Integer Linear
Programming approach (MILP) (Wolsey, 1998) to develop a mathematical model of a

workforce scheduling environment. A number of researchers including Danzig (1954) proposed MILP models for workforce scheduling. Difficulties with the use of the prior art MILP models for workforce scheduling are well documented (Nanda and ~~Drowne~~ Browne, 1992, page 206, Holloran and Byrn, 1986, page 13). The MILP model proposed by Dantzig (1954), for example, enumerates all possible combinations of shift and tour parameters resulting in tens of trillions of decision variables, making it very inefficient and impossible to solve even in small contact center environments. To overcome these limitations of the prior art MILP agent scheduling models, the present invention formulates daily break, and days-off scheduling implicitly (i.e. not explicitly enumerating all possible combinations). Thus, the method of the present invention formulates a significantly smaller but equally powerful MILP model, and uses various extensions of it.

Please replace paragraph 1 on page 9 with the following amended paragraph:

The method of the present invention first formulates ~~an~~ a skills based or non-skills based agent scheduling environment involving a plurality of agent skill and work groups, and plurality of contact types as an MILP model including various contact center constraints and requirements. The formulation step of the present invention involves, first, the definitions of various decision variables, and parameters, and the formulation of an objective function. The objective function of the MILP model of the present invention determines a measure of effectiveness (or merit) for each solution. Examples of the objective function in the method include total schedule cost, total time scheduled, total paid time scheduled, total agent preference, etc. The formulation step also includes the expression of contact center constraints, and agent and skill requirements in all periods to be scheduled as mathematical equalities or inequalities. Finally, constraints that restrict the values of all decision variables to nonnegative values, and some variables to nonnegative integer values only are added to the formulation.

Please replace paragraph 1 on page 11 with the following amended paragraph:

If the best known (integer feasible) solution was changed in the RA algorithm, the B&C algorithm updates the best integer solution known for its own use, and forms new sub-problems (nodes) by adding new constraints (Wolsey, 1998). The entire process is then repeated; the B&C algorithm chooses a new sub-problem, and solves its LP relaxation. If the LP relaxation has a feasible solution satisfying all of the constraints but some of the integrality constraints for some decision variables, the B&C algorithm transfers control to the RA algorithm which searches for an integer feasible solution through rounding and adding more agents to the schedules. The solution algorithm of the present invention terminates when all sub-problems in the B&C algorithm are terminated (for sub-problem termination conditions in the B&C algorithm are given in Wolsey, 1998). That is when the optimality condition is satisfied by an integer feasible solution to the MILP model. When this condition is satisfied, control is transferred to the schedule-reporting step.

Please replace lines 10-11 on page 12 with the following amended lines:

FIG. 4A, FIG. 4B and FIG. 4C show FIG. 4 shows a flowchart of the solution method for the MILP model of the invention.

Please replace lines 26-31 on page 13 with the following amended lines:

scheduling environment parameters provided in module (3). The present invention provides a method for the MILP model generation for a workforce scheduling environment, and an algorithm for the a near-optimal or optimal solution of the MILP model generated by the MILP model generator to develop optimal agent schedules. The method of the present invention covers both single-skill non-skills based and skills based agent scheduling environments.

Please replace lines 12-18 on page 15 with the following amended lines:

~~A number of formulations for agent scheduling in an environment involving agents with a single skill and single contact type, and their extensions are discussed first to disclose various modeling techniques used in the MILP model generation method of the invention.~~ A number of MILP formulations for agent scheduling in a non-skills based environment (also referred to as "single skill environment" in the rest of this patent document) and their extensions are discussed first. The MILP model generation method of the present invention for skills based agent scheduling environments involving agents with ~~different~~ multiple skills serving multiple contact types is disclosed after this discussion. The method of the invention for ~~optimal~~ the solution of the MILP model is disclosed thereafter.

Please add the following new paragraph after paragraph 4 (lines 12-18) on page 15:

To facilitate the description of the MILP model and its extensions used by the present invention in this patent document, a variable or set defined with a subscript is shown as flat text when the variable or set itself is a subscript of another term. For instance, a set defined as $B1_{kih}$ is typed as $_{B1kih}$ in $\sum_{t \in B1kih}$ where it is a subscript of the summation sign $\sum$. Moreover, symbols "ε" and "$\in$" are both used to imply membership in a set or indicate an element of a set.

Please replace lines 22-26 on page 16 with the following amended lines:

Let K be the set of all distinct tour types available to assign agents. Each tour type k has a predetermined set of start times given in I_k. . Let a_{kiht} be equal to one if period t on day h is in the ~~tour~~ shift span (that is, a work, break, or days-off period) of tour k with a daily start time of i, and zero otherwise. Suppose that the maximum number of agents available to assign to tour k is limited to D_k^{max}. Let X_{ki} be the number of agents assigned

Please add the following paragraph after line 10 on page 18 as amended paragraph 1:

An MILP formulation (the terms "model" and "formulation" are used to have the same meaning in the remainder of this patent document) consists of decision variables, objective function, constraints, and parameters as in (1-8) above. Decision variables represent various activities, and decisions regarding the values they should take. One objective in formulating an MILP model such as (1-8) is to determine the values for decision variables that minimize (or maximize in some other MILP models) the objective function, subject to constraints. Values a decision variable can take are limited by the constraints of the MILP formulation. A set of values for all decision variables satisfying all constraints of an MILP formation is called an integer feasible solution to the formulation. An integer feasible solution that provides the best value for the objective function of an MILP over all possible integer feasible solutions is called an optimal solution (Taha, 1987). In the MILP formulation (1-8), decision variables include $X_{ki}, Y_{kih}, U_{kiht}, W_{kiht}$ and V_{kiht} for all k, i, h, and t. Constraints are given in (2-8), and the objective function in (1). The terms decision variables, constraints, objective function, feasible solution and optimal solutions are used to carry the meanings stated above, although different MILP models and extensions may have different sets of decision variables, constraints, and objective functions. When all integrality constraints in an MILP formulation (e.g. constraints (8) above) are relaxed, the resulting model is referred to as the Linear Programming relaxation ("LP Relaxation"). A set of values satisfying all constraints except the integrality constraints of an MILP is a feasible solution to its LP relaxation. An optimal solution to the LP relaxation may or may not satisfy the integrality constraints of the original MILP model. If an optimal solution to the LP relaxation of an MILP also satisfies all of its integrality constraints, it is also an optimal solution to the MILP (Taha, 1987).

Please replace lines 6-7 on page 20 with the following amended lines:

~~Demand constraint~~ Constraint (2) is then formulated by adding O_{ht} for overstaffing and S_{ht} for shortages to its left side as follows:

Please replace lines 20-22 and equation (11a) on page 20 with the following amended lines and equation:

If a certain amount of overstaffing is needed for other tasks (e.g. to answer email or fax based contacts), it can be ensured by restricting the overstaffing to be greater than or equal to the desired level ~~f_h~~ $\underline{f_{ht}}$.

$$[[\textstyle\sum_{t \varepsilon Th} O_{ht} \geq f_h ,]] \quad \textstyle\sum_{t \varepsilon Th} O_{ht} \geq f_{ht} , \qquad t\varepsilon T_h, \ h= 1,\ldots,7, \tag{11a}$$

Please replace equation (11b) on page 20 with the following amended equation where d_h is corrected as d_{ht}:

$$[[\textstyle\sum_{t \varepsilon Th} S_{ht} \geq d_h ,]] \quad \textstyle\sum_{t \varepsilon Th} S_{ht} \geq d_{ht} , \qquad t\varepsilon T_h, \ h= 1,\ldots,7, \tag{11b}$$

Please replace lines 1-2 on page 21 with the following amended lines:

where ~~d_h~~ $\underline{d_{ht}}$ is the maximum understaffing tolerated on day h. Constraints (11a) and (11b) can easily be further modified to impose limits for a specific period(s) on day h.

Please replace lines 1-2 on page 21 with the following amended lines where d_h is corrected as d_{ht}:

where d_{ht} is the maximum understaffing tolerated on day h. Constraints (11a) and (11b) can easily be further modified to impose limits for a specific period(s) on day h.

Please replace the last paragraph on page 21 with the following amended paragraph:

In MILP model (1-8), even though daily shift start times for an agent remain the same on different days, break times may vary from one work day to another. For instance, it is possible to have the first relief break of an agent be scheduled at 9:00am on Monday, and at 10:00am on Tuesday. If a tour (say tour k) requires consistent relief and lunch break times on all work days and <u>for all agents starting at start time i to be on break at the same time</u> in addition to consistent daily shift start times and shift lengths, these requirements are ensured by adding the following constraints the MILP model (1-8):

Please replace equations (12a-12c) and (13a-13c) on page 22 with the following amended equations in which "=" is replaced by "≤" (please note – in the case of the greater than or equal to sign, "≤", underlining does not work in Word):

$$[[\textstyle\sum_h U_{kiht} = q_k\, SU_{kit}\,,]] \qquad \underline{\textstyle\sum_h U_{kiht} \le q_k\, SU_{kit}}\,, \qquad i\epsilon I_k,\; t\epsilon D1_{ki}\,, \quad (12a)$$

$$[[\textstyle\sum_h W_{kiht} = q_k\, SW_{kit}\,,]] \qquad \underline{\textstyle\sum_h W_{kiht} \le q_k\, SW_{kit}}\,, \qquad i\epsilon I_k,\; t\epsilon D2_{ki}\,, \quad (12b)$$

$$[[\textstyle\sum_h V_{kiht} = q_k\, SV_{kit}\,,]] \qquad \underline{\textstyle\sum_h V_{kiht} \le q_k\, SV_{kit}}\,, \qquad i\epsilon I_k,\; t\epsilon D3_{ki}\,, \quad (12c)$$

$$\textstyle\sum_{t\epsilon D1ki} SU_{kit} = 1, \qquad\qquad i\epsilon I_k,\; \underline{k\epsilon K,} \qquad\qquad (13a)$$

$$\textstyle\sum_{t\epsilon D2ki} SW_{kit} = 1, \qquad\qquad i\epsilon I_k,\; \underline{k\epsilon K,} \qquad\qquad (13b)$$

$$\textstyle\sum_{t\epsilon D3ki} SV_{kit} = 1, \qquad\qquad i\epsilon I_k,\; \underline{k\epsilon K,} \qquad\qquad (13c)$$

Please replace lines 11-16 on page 22 with the following amended lines:

where $D1_{ki}$, $D2_{ki}$, and $D3_{ki}$ are the sets containing the ~~first~~ consistent (across the days of a week) daily first relief, lunch, and second break times in $B1_{kih}$, $B2_{kih}$, and $B3_{kih}$,

respectively, q_k ~~is the number of work days tour k requires~~ is a very large number, SU_{kit}, SW_{kit}, and SV_{kit} are binary variables taking a value of either zero or one to indicate whether period t is selected as the consistent daily first relief, lunch, or second relief break time, respectively.

Please replace paragraph 2 (lines 11-15) on page 27 with the following amended paragraph:

To facilitate the discussion and notation, the scheduling environment described before the disclosure of MILP model (1-8) is considered again. The present invention also extends the MILP model to tours not requiring consistent daily shift start times and same daily shift length requirements. For these tours, ~~both the~~ shift start time as well as shift duration for an agent may vary from one day to another.

Please replace paragraph 3 (lines 17-26) on page 27 with the following amended paragraph:

Assume that the scheduling environment involves only tours not requiring consistent daily start time and shift length. Let QK be the set of all such tour types. Assume that agents assigned to tour type k can start at any one of a set of predetermined start times ~~for day h~~ given in ~~QI_{kh}~~ Q_k. Let the daily shift lengths allowed for tour type k be F_k. F_k may also change from one day to another. Note that tour k has a minimum weekly work limit specified by the shortest shift in F_k, and a maximum weekly work limit specified by the longest shift in F_k. The daily shift start times should not allow for back-to-back shift schedules for agents without allowing enough time for rest. Thus, the latest start time for a tour plus the length of the longest daily shift type are not allowed to exceed 24 hours minus the desired minimum rest period between consecutive daily shifts. To prevent this for a tour type, the present invention creates separate tour types each with start times allowing a minimum required time between the latest possible end of a shift on one day and the earliest possible start of a shift on the following day.

Please replace lines 12-14 on page 28 with the following amended lines:

start period, respectively. Let a_{kniht} be equal to one if period t on day h is in the shift span (that is, a work or a break/days-off period) of agents assigned to tour k who have the start time of i and shift length n on day h, and zero otherwise. A work pattern in this

Please replace lines 17-19 on page 28 with the following amended lines:

Let the set of all allowed work and non-work day patterns for the agents assigned to tour k be QL_k. Only the work patterns in FIG. 3 satisfying the work and non-work day rules specified for tour k are included in QL_k (e.g.

Please replace constraint (41) on page 29 with the following amended lines:

QX_{knhi}, Q_{kl}, QU_{kniht}, QW_{kniht}, and QV_{kniht} ≥ 0 and integer for all k, n, i, h and t.   (41)

Please replace paragraph 2 on page 30 with the following amended paragraph:

Objective function (34) can be easily reformulated to minimize the total amount of agent time scheduled (or paid agent time) scheduled by setting C_{kl} equal to scheduled time (or paid agent time) per agent assigned to tour k and work pattern l, $l \epsilon QL_k$, and $k \epsilon QK$. Similarly, if agents rate their most preferred, second most preferred, etc., tours using an RAting a rating system, these ratings for various tours and work patterns can be aggregated and used in the objective value to maximize agent preference in scheduling tours and work patterns. Note that in this case, in In order to not to schedule redundant tours and increase agent costs unnecessarily, rating system should assign the lowest value to the most preferred tour, and next value higher to the second most preferred tour, etc. Consequently, the objective function will remain as of minimization type to the maximize agent preference.

Please replace last paragraph on page 31 with the following amended paragraph (subscript t is added to f_{ht}):

If a certain amount of overstaffing is needed for other tasks (e.g. to answer email or fax based contacts), it can be assured by restricting the overstaffing to be greater than or equal to the desired level ~~f_h~~ f_{ht}.

Please replace constraint (45) on page 32 with the following amended constraint (46) (subscript t is added to f_{ht}):

$$[[\sum\nolimits_{t \in QTh} O_{ht} \geq f_h,]] \underline{\sum\nolimits_{t \in QTh} O_{ht} > f_{ht}}, \qquad\qquad t \epsilon T_h, \ h= 1,\ldots,7, \qquad (45)$$

Please replace constraint (46) and lines 9-10 on page 32 with the following amended constraint (46) (subscript t is added to d_{ht}) and lines 9-10:

$$[[\sum\nolimits_{t \in QTh} S_{ht} \geq d_h,]] \underline{\sum\nolimits_{t \in QTh} S_{ht} > d_{ht}}, \qquad\qquad t \epsilon T_h, \ h= 1,\ldots,7, \qquad (46)$$

where ~~d_h~~ d_{ht} is the minimum overtime desired on day h. Note that (45) and (46) can easily modified to impose limits for a specific period(s) on day h.

Please replace paragraph 1 on page 33 with the following amended paragraph:

If working hours are shorter on certain days for a tour or a contact center, the present invention extends the MILP model by excluding the daily shift variables that has a late start time resulting an end time after the early closure time for that tour or center. ~~Note that break~~ Break variables as well as break constraints (34-41) associated with these shift variables are excluded.

Please replace paragraph 2 and constraint (47) on page 34 with the following amended paragraph and constraint (47):

Another way that the present invention extends the MILP model is to merge formulations (1-8) and (34-41) for this type of scheduling environments to obtain optimal schedules. To distinguish cost parameters, Cx^j_{ki} is introduced as the cost of assigning an agent to X_{ki}. Using the definitions of the parameters and variables given for formulations (1-8) and (34-41), merging the objective functions (1) and (34) and demand constraints (2) and (35), and including the necessary break and days-off constraints from formulations (1-8) and (34-41), the present invention extends the MILP model for a scheduling environment involving both consistent and non-consistent shift start times and shift lengths as follows:

$$\text{Minimize} \quad [[\Sigma_{k \in K} \Sigma_{i \in Ik} \; C_{ki} \, X_{ki}]] \; \underline{\Sigma_{k \in K} \Sigma_{i \in Ik} \; Cx_{ki} \, X_{ki}} + \Sigma_{k \in QK} \Sigma_{l \in Ql_k} C_{kl} \, Q_{kl}$$

$$+ \Sigma_{k \in QK} \Sigma_{ns Fk} \Sigma_h \Sigma_{i \in Qlk} \, c_{kni} \, QX_{knhi} \qquad (47)$$

Please replace constraint (48) starting on page 35 and continuing to page 36 with the following amended constraint (48) (with Qik corrected as QIk in the third line and first term of the constraint):

$$\Sigma_{k \in K} \Sigma_{i \in Ik} \, a_{kiht} \, X_{ki} \; - \; \Sigma_{k \in K} \Sigma_{i \in Ik} \, a_{kiht} \, (Y_{kih} + Y_{ki(h-1)}) \; - \; \Sigma_{k \in K} \Sigma_{i \in T1kht} \, U_{kiht}$$

$$- \; \Sigma_{k \in K} \Sigma_{i \in T2kht} \, (W_{kiht} + W_{kih(t-1)}) \; - \; \Sigma_{k \in K} \Sigma_{i \in T3kht} \, V_{kiht}$$

$$[[+ \Sigma_{k \in QK} \Sigma_{ns Fk} \Sigma_{i \in Qik} \, a_{kniht} \, QX_{knhi}]] \; \underline{+ \Sigma_{k \in QK} \Sigma_{ns Fk} \Sigma_{i \in QIk} \, a_{kniht} \, QX_{knhi}} \; - \; \Sigma_{k \in QK} \Sigma_{ns Fk}$$

$$\Sigma_{i \in QT1knht} \, QU_{kniht}$$

$$- \Sigma_{k \in QK} \Sigma_{n \varepsilon Fk} \Sigma_{i \varepsilon QT2knht} (QW_{kniht} + QW_{knih(t-1)})$$

$$- \Sigma_{k \in QK} \Sigma_{n \varepsilon Fk} \Sigma_{i \varepsilon QT3knht} QV_{kniht} \geq b_{ht}$$

$$t \varepsilon T_h, \ h = 1, \ldots, 7$$

$$(48)$$

Please replace constraint (59) on page 35 with the following amended constraint (59) ("n" added between k and i):

X_{ki}, Y_{kih}, QX_{knhi}, Q_{kl}, U_{kiht}, W_{kiht}, and V_{kiht}, QU_{kniht}, QW_{kniht}, and QV_{kniht} ≥ 0 and
integer for all k, n̲, i, h, and t, (59)

Please replace lines 1-9 on page 36 with the following amended lines:

where objective function (47) combines objectives functions (1) and (34), constraint (48) ~~combined~~ combines agent, break, and days-off variables specifying agent availability in a period from constraints (2) and (35), constraints (49-51) are the break balance constraints for the tours requiring consistent daily shift start times and shift lengths, constraint (52) ensures that sufficient number of days-off are scheduled for the agents assigned to tours requiring consistent daily shift start times and shift lengths, constraints (53-55) ~~ensures~~ ensure that sufficient number of first, lunch, and second relief breaks are scheduled for the agents assigned to ~~tour~~ tours not requiring consistent daily shift start times and shift lengths, constraint (56) ensures that the number of shifts scheduled on a given day is equal to the number agents who are assigned to work on that day by the work patterns scheduled, constraints (57) and (58) ensure that the numbers of agents assigned to various tours requiring consistent or non-consistent daily shift start times and shift lengths don't exceed the limits on the agent availability for these tours. Finally, constraint (59) restricts the tour, days-off, daily shift, work pattern, and break variables to nonnegative integers to avoid negative values and/or fractional agent schedules.

Please replace lines 29-31 on page 36 with the following amended lines:

The present invention applies the MILP model to scheduling environments involving a plurality of agent skills groups, agent work groups, and contact types requiring different agent skills.   An agent skill group contains agents with the same skill sets <u>and proficiency levels (also referred to as "skill levels")</u>.

Please replace lines 1-5 on page 37 with the following amended lines:

sets. Let R be the set of different contact types with known agent requirements in all periods and days. Let the number of agents with qualified skills for contact type rεR required in period t on day h be b^r_{ht}. Suppose now that there are a number of agent skill groups jεJ, each with a unique set of skills. Let the set of contact types that ~~agent~~ <u>skills</u> group j is qualified to provide service be Nj, jεJ. <u>Agent requirements should reflect the multi-skill efficiency expected in a skills-based routing environment due to agents with multiple skills and, depending on the scheduling environment, may be specified for skills groups rather than contact types.</u> A contact type may be served by multiple agent groups. Let the set of ~~agent skill~~ <u>skills</u> groups that can service contact type r be M^r.

Please replace constraint (61) starting on page 37 with the following amended constraint (61) (with "+" in front of the 5th term replaced by "-"):

$$\sum_{k \in QKj} \sum_{n \varepsilon Fk} \sum_{i \varepsilon Qlk} a_{knhi} QX^j_{knhi} - \sum_{k \in QKj} \sum_{n \varepsilon Fk} \sum_{i \varepsilon QT1knht} QU^j_{kniht}$$

$$- \sum_{k \in QKj} \sum_{n \varepsilon Fk} \sum_{i \varepsilon QT2knht} (QW^j_{kniht} + QW^j_{knih(t-1)})$$

$$- \sum_{k \in QKj} \sum_{n \varepsilon Fk} \sum_{i \varepsilon QT3knht} QV^j_{kniht} \quad [[+ \sum_{r \in Nj} G^{jr}_{ht}]] - \sum_{r \in Nj} G^{jr}_{ht} = 0,$$

$$\underline{j \varepsilon J}, t\varepsilon T_h, h = 1, \ldots, 7, \qquad (61)$$

Please replace line 25 on page 38 with the following amended line:

notation introduced for (1-61) is now extended to distinguish agent ~~skills~~ <u>skill</u> groups. For

Please replace lines 28-29 on page 37 with the following amended lines:

agents who are scheduled to be on a relief or lunch break during this period. Thus, the difference is equal to the number of agents who are scheduled and working <u>(available)</u> during period

Please replace constraint (63) starting on page 39 with the following amended constraint (63) (with "+" in front of the 2nd term replaced by "-"):

$$f^j_{ht}(X, Y, Z, U, W, V, QX, Q, QU, QW, QV) \; [[, + \Sigma_{r \in Nj} \;\; G^{jr}_{ht} \;]] - \Sigma_{r \in Nj} \;\; G^{jr}_{ht} \;\; = \; 0,$$

$$, j = 1, ..., n \;, t \varepsilon T_h, \; h = 1,...,7, \qquad (63)$$

Please replace lines 14-21 on page 39 with the following amended lines:

where e^{jr} is the relative efficiency of an agent from agent group j in serving contact type r with respect to an agent whose primary skill <u>(highest proficiency level)</u> is serving contact type r, $e^{jr} \varepsilon [0, 1]$. For instance, if $e^{jr} = 1$, agent group j is equally efficient, and if $e^{jr} = 0.75$, agents in group j are 75% efficient with respect to an agent whose primary skill is serving contact type r. ~~Because of call blending (mixing the calls routed to an agent in a planning period), values~~ Values of G^{jr}_{ht} variables do not have to be integer. ~~But their sums in each period t, day h, and for agent group j will be integers since the variables in $f^j_{ht}(.)$ in constraint (63) are restricted to integer values.~~

Please replace lines 26-29 on page 39 with the following amended lines:

groups, remain the same since the agent allocation does not affect the break and days-off scheduling constraints. Objective functions (14) and (34) are merged ~~ands~~ and modified to include potential wage differentials for different agent groups. The MILP model of the present invention is now disclosed below.

Please replace objective function (65) starting on page 40 with the following amended objective function (65) ("$\underline{\Sigma_{j \in J}}$" is added and "$C^j_k$" is changed to "$Cx^j_k$"):

$$\text{Minimize} \quad \underline{\Sigma_{j \in J}}\Sigma_{i \in I_k} \ \Sigma_{k \in K_j} \ Cx^j_{ki} \ X^j_{ki} \ + \ \underline{\Sigma_{j \in J}} \ \Sigma_{l \in QL_k} \Sigma_{k \in QK_j} \ C^j_{kl} \ Q^j_{kl}$$

$$+ \ \underline{\Sigma_{j \in J}} \ \Sigma_{k \in QK_j} \Sigma_{n \in F_k} \Sigma_h \Sigma_{i \in Ql_k} \ c^j_{kni} \ QX^j_{knhi} \quad (65)$$

Please replace constraint (67) on page 40 with the following amended constraint (67) (with "+" in front of the 2nd term replaced by "-"):

$$f^j_{ht}(X, Y, Z, U, W, V, QX, Q, QU, QW, QV)$$

$$[[+ \Sigma_{r \in Nj} \ G^{jr}_{ht}]] - \Sigma_{r \in Nj} \ G^{jr}_{ht} \ = \ 0$$

$$, j \in J \ , t \in T_h, \ h = 1,\ldots,7, \quad (67)$$

Please replace constraint (68-74) on page 40 with the following amended constraint (68-74) (", j∈J" is added):

$$X^j_{ki} - Y^j_{kih} - Y^j_{ki(h-1)} - \Sigma_{m \neq h,(h-1)} Z^j_{kimh} = \Sigma_{t \in Blkih} U^j_{kiht} \quad \underline{, j \in J}, i \in I_k, \ k \in K_j, \ h = 1,\ldots,7, \quad (68)$$

$$X^j_{ki} - Y^j_{kih} - Y^j_{ki(h-1)} - \sum_{m \neq h,(h-1)} Z^j_{kimh} = \sum_{t \in B2kih} W^j_{kiht} \quad \text{j} \in \text{J}, i \in I_k, k \in K_j, h = 1,...,7, \quad (69)$$

$$X^j_{ki} - Y^j_{kih} - Y^j_{ki(h-1)} - \sum_{m \neq h,(h-1)} Z^j_{kimh} = \sum_{t \in B3kih} V^j_{kiht} \quad \text{j} \in \text{J}, i \in I_k, k \in K_j, h = 1,...,7, \quad (70)$$

$$r_k Y^j_{kih} = \sum_{l \neq h,(h+1)} Z^j_{kihl} \quad \text{j} \in \text{J}, i \in I_k, k \in K_j, h = 1,...,7, \quad (71)$$

$$Z^j_{kihm} \leq Y^j_{kih} \quad \text{j} \in \text{J}, i \in I_k, k \in K_j, h = 1,...,7,$$
$$m \neq h, (h+1) \quad (72)$$

$$X^j_{ki} = \sum_h Y^j_{kih} \quad \text{j} \in \text{J}, i \in I_k, k \in K_j, \quad (73)$$

$$QX^j_{knhi} = \sum_{t \in QB1knih} QU^j_{kniht} \quad \text{j} \in \text{J}, n \in F_k, i \in QI_k, k \in QK_j, h = 1,...,7, \quad (74)$$

Please replace constraint (75-77) on page 41 with the following amended constraint (75-77)
(", jεJ" is added):

$$QX^j_{knhi} = \sum_{t \in QB2knih} QW^j_{kniht} \quad \text{j} \in \text{J}, n \in F_k, i \in QI_k, k \in QK_j, h = 1,...,7, \quad (75)$$

$$QX^j_{knhi} = \sum_{t \in QB3knih} QV^j_{kniht} \quad \text{j} \in \text{J}, n \in F_k, i \in QI_k, k \in QK_j, h = 1,...,7, \quad (76)$$

$$\sum_{l \in QLk} A_{klh} Q^j_{kl} = \sum_{n \in Fk} \sum_{i \in Ik \; i \in QIk} QX^j_{knhi} \quad \text{j} \in \text{J}, k \in QK_j, h = 1,...,7, \quad (77)$$

Please replace constraint (80) starting on page 41 with the following amended constraint (80)
("n" added between k and i):

$$X^j_{ki}, Y^j_{kih}, Z^j_{kihm}, U^j_{kiht}, W^j_{kiht}, V^j_{kiht}, QX^j_{knhi}, Q^j_{kl}, QU^j_{kniht}, QW^j_{kniht}, QV^j_{kniht} \geq 0 \text{ and}$$
integer for all j, k, n, i, h, and t, and $G^{jr}_{ht} \geq 0$ for all j, r, h, and t. (80)

Please replace lines 18-22 on page 42 with the following amended lines:

G^{jr}_{ht} in formulation (65-80) above doesn't have to be integer even though it represents the number of agents from group j allocated to serve type r contacts. ~~This is due to the fact that ACD's may blend contact types to have an agent serve multiple contact types in a planning period.~~ Hence, formulation (65-80) is a Mixed Integer Programming (MILP) formulation involving both integer and continuous variables.

Please replace lines 10-22 on page 42 with the following amended lines:

The present invention also extends the MILP model to obtain a second formulation by defining pseudo tours for the tours requiring consistent daily shift start times as follows. Consider a tour type, say tour k, $k \in K_j$, available for ~~agent~~ skill group j. ~~The set~~ Set K_j contains the valid tour types for ~~agent~~ skill group j that require consistent daily shift start times. Constraints (68-73), (78), and six terms ~~in~~ on the right side of (62) involving variables X^j_{ki}, Y^j_{kih}, Z^j_{kih}, U^j_{kiht}, W^j_{kiht}, and V^j_{kiht} specify the break placement and days off scheduling requirements and agent availability for tours in K_j. The present invention extends the MILP model for the second formulation by first considering all valid ~~days-off~~ work patterns given in FIG. 3 and identifying the patterns satisfying constraints (71-73) for $k \in K_j$. ~~These constraints thus become redundant (i.e. not included).~~ A distinct pseudo tour is then defined for every daily start time specified in I_k, $k \in K_j$, with the valid work patterns identified. Let one such pseudo tour be α. Then, α has only one daily start time $\underline{\text{in}}$ OI_α representing one of the daily start times in I_k, has the

Please replace objective function (81) starting on page 43 with the following amended objective function (81) ("$\underline{\Sigma_{j \in J}}$" is added):

Minimize $\underline{\Sigma_{j \in J}}$ $\Sigma_{l \in QLk}$ $\Sigma_{k \in QKj}$ C^j_{kl} Q^j_{kl} + $\underline{\Sigma_{j \in J}}$ $\Sigma_{k \in QKj}$ $\Sigma_{n \in Fk}$ Σ_h $\Sigma_{i \in QIk}$ c^j_{kni} QX^j_{knhi} (81)

Please replace constraint (83) on page 43 with the following amended constraint (83) (with "+" in front of the 2nd term replaced by "-"):

$$f^j_{ht}(QX, Q, QU, QW, QV) \; [[+ \textstyle\sum_{r \in Nj} \; G^{jr}_{ht}]] - \textstyle\sum_{r \in Nj} \; G^{jr}_{ht} \; = \; 0 \quad , j \epsilon J, t \epsilon T_h, h = 1,\ldots,7,$$

$$(83)$$

Please replace constraint (84-87) on page 43 with the following amended constraint (84-87) (", jɛJ" is added):

$$QX^j_{knhi} = \textstyle\sum_{t \in QB1knih} QU^j_{kniht} \qquad , j\epsilon J, n\epsilon Fk, i\epsilon QI_k, k \epsilon QK_j, h = 1,\ldots,7, \quad (84)$$

$$QX^j_{knhi} = \textstyle\sum_{t \in QB2knih} QW^j_{kniht} \qquad , j\epsilon J, n\epsilon Fk, i\epsilon QI_k, k \epsilon QK_j, h = 1,\ldots,7, \quad (85)$$

$$QX^j_{knhi} = \textstyle\sum_{t \in QB3knih} QV^j_{kniht} \qquad , j\epsilon J, n\epsilon Fk, i\epsilon QI_k, k \epsilon QK_j, h = 1,\ldots,7, \quad (86)$$

$$\textstyle\sum_{l \epsilon QLk} A_{klh} Q^j_{kl} = \textstyle\sum_{n\epsilon Fk} \textstyle\sum_{i\epsilon Ik \; i\epsilon QIk} QX^j_{knhi} \qquad , j\epsilon J, k \epsilon QK_j, h = 1,\ldots,7, \qquad (87)$$

Please replace constraint (89) on page 43 with the following amended constraint (89) ("n" added between k and i):

QX^j_{knhi}, Q^j_{kl}, QU^j_{kniht}, QW^j_{kniht}, and $QV^j_{kniht} \geq 0$ and integer for all j, k, n, i, h, t, and $G^{jr}_{ht} \geq 0$ for all j, r, h, and t. (89)

Please replace constraint (90) starting on page 43 and continuing on to page 44 with the following amended constraint (90):

$[[f^j_{ht}(X, U, W, V, Y, Z, QX, Q, QU, QW, QV)]] \; f^j_{ht}(QX, Q, QU, QW, QV) =$

$$\sum_{k \in QKj} \sum_{n \varepsilon Fk} \sum_{i \varepsilon QIk} a_{kniht} \, QX^j_{knhi}$$

$$- \sum_{k \in QKj} \sum_{n \varepsilon Fk} \sum_{i \varepsilon QT1knht} QU^j_{kniht}$$

$$- \sum_{k \in QKj} \sum_{n \varepsilon Fk} \sum_{i \varepsilon QT2knht} (QW^j_{kniht} + QW^j_{knih(t-1)})$$

$$- \sum_{k \in QKj} \sum_{n \varepsilon Fk} \sum_{i \varepsilon QT3knht} QV^j_{kniht}$$

$$, j \varepsilon J, \, t \varepsilon T_h, \, h = 1, \ldots, 7, \quad (90)$$

Please replace paragraph 1 (lines 9-20) on page 44 with the following amended paragraph:

Extension of formulations (65-80) and (81-89) to the cases discussed in relation to formulations (1-8) and (34-41) is straightforward. ~~It~~ For example, it should be clear to those skilled in the art that the information disclosed in relation to formulations (1-8–) and (34-41) can be readily utilized as a basis to extend formulations (65-80) and (81-89) of the present invention to the cases involving (i) minimizing scheduled/paid time or maximizing agent preference, (ii) schedule generation when there aren't enough agents (i.e. there are agent shortages in some periods for some contact types), (iii) tours with longer and/or more/less breaks, (iv) fixed days-off, and (v) days with a closure time earlier than the other days. In yet another embodiment, agent requirements may be given for skill group instead of contact types. Let b^j_{ht} be the number of agents from skill group j needed in period t on day h. In this case, constraints (82) and (83) are combined as follows.

$$f^j_{ht}(QX, Q, QU, QW, QV) + S^r_{ht} - O^r_{ht} = b^j_{ht} \quad , j \varepsilon J, \, t \varepsilon T_h, \, h = 1, \ldots, 7, \quad (91)$$

With constraint (91) replacing (82) and (83), the MILP formulation (81-89) can be separated into a series MILP formulations, resulting in one MILP formulation for each skills group j, jεJ. Accordingly, it is to be understood that the embodiments of the

invention herein described are merely illustrative of the application of the principles of

the invention. Reference herein to details of the illustrated embodiments are not intended

to limit the scope of the claims, which themselves recite those features regarded as

essential to the invention.

Please replace lines 27-30 on page 44 with the following amended lines:

Formulations (65-80), and (81-89), and their extensions disclosed before can be

~~solve~~ solved to obtain a global optimal solution in a number of ways. Such a global

optimal solution satisfies the necessary and sufficient conditions for optimality of the

MILP model of the invention. The solution method of the invention will be discussed in

the context of formulations (65-80~~,~~) and (81-90).

Please replace the 2nd paragraph on page 45 with the following amended paragraph:

The branch and cut (B&C) algorithm is a well known technique for optimally

solving MILP formulations such as (65-80), and (81-89) (Wolsey, 1998). A computer

program implementing the B&C algorithm can be developed using any programming

language or can be obtained from a number of software companies including Ilog, Inc.,

and Dash Optimization. These providers are also offering their B&C algorithm in the

form of a callable library that can be called from an integrated MILP model ~~generate~~

generator and optimizer module. In either case, the B&C algorithm forms a part of the

solution method of the present invention.

Please replace lines 17-31 on page 46 with the following amended lines:

The B&C algorithm first solves the continuous version (i.e. Linear ~~Programmiing~~

Programming (LP) relaxation) of an MILP model (Wolsey, 1998) (step (17)). This node

is called node zero (subsequent nodes are numbered sequentially). If the continuous

solution thus found is integer feasible and optimal (step (20)), then the optimizer

terminates with an optimal solution to the MILP model (step (21)). If this solution is not

integer feasible (i.e. if some integer restricted variables have non-integer values), then the
B&C algorithm starts creating nodes (sub-problems) by adding constraints on certain
decision variables to eliminate non-integer values. In this case, the objective value of the
solution of the LP relaxation is a lower bound on the lowest objective value that can ever
be achieved for the scheduling environment formulated in the MILP model. That is, no
solution to the MILP model can have an objective value better than the lower bound thus
obtained. ~~Before solving the LP relaxation at node zero, the lower bound is positive
infinity if no solution is known beforehand.~~ After solving the LP relaxation at node 0, if
the solution is a feasible solution to the LP relaxation but violates some integrality
constraints (step **(22)**), the B&C algorithm calls the RA algorithm. If the RA algorithm
locates an integer feasible

Please replace step 31 on page 48 and continuing on to page 49 with the following amended
step:

Step **31**: Compute the <u>scheduled</u> agent availability in each period for all agent groups
(i.e. $f^j_{ht}(X, U, W, V, QX, Y, Q, QU, QW, QV)$ in equation (67)). Compare the
number of agents available in each agent group j in a planning period, say tϵT$_h$ on
day h, with the values of the agent allocation variables G associated with this
agent group in planning period t. If the number of agents available to work for
agent group j is ~~greater than~~ <u>not equal to</u> the sum of the current values of the agent
allocation variables (from step **25**) for all contact types in N$_j$ in period t on day h,
~~allocate the excess agents~~ <u>adjust values of associated allocation variables</u> to meet
the requirements by contact types in N$_j$. Prioritize contact types in each period by
(i) maximum shortage, (ii) whether this agent

Please replace step 34 and 35 on page 49 with the following amended step:

Step **34:** If a feasible schedule <u>satisfying all agent requirements</u> is found either by the
B&C algorithm, or in the earlier calls to the RA algorithm, go to step **35**.
Otherwise, go to step **36**.

Step **35**: Discard the current infeasible solution (that is, an integer solution with agent shortages). Stop the RA algorithm, and return control back to the B&C algorithm (step **(17)**).

Please replace step 36 on page 46 with the following amended step:

Step **36**: If an integer feasible solution to the MILP model hasn't been found by the B&C and RA Algorithms at earlier tries, compare the new infeasible solution with the infeasible solution recorded in earlier calls to the RA algorithm, and update the best infeasible solution if the new one has fewer period shortages. If this is node zero, store the infeasible solution if there are no known solution. ~~Otherwise, stop~~ Stop the RA algorithm, and return back to the B&C algorithm (step **(17)**).

Please replace step 37a on page 50 with the following amended step:

37a: For each agent skill group j with available agents (i.e. the number of agents already scheduled in the rounded solution found in step **(25)** is less than the maximum number of agents available for ~~each~~ an agent skill group), consider tour type k that is available for this agent group. If the tour requires consistent daily start time and shift length, that is kεK$_j$, go to **37b**. Otherwise, kεQK$_j$ and go to **37c**.

Please replace step 37e on page 51 and continuing on to page 52 with the following amended step:

37e: If tour k requires consistent daily shift start times and lengths, compare the peak shortage covered by tours starting at different times in Γ^i_k, ~~kεK$_j$,~~ and find the start time i* with the highest peak shortage covered. If there are ties between two or more start times in Γ^i_k, select the one with the highest shortage coverage per dollar

(total shortage coverage) / C^j_{ki}

If tour k does not require consistent daily shift start times and lengths, compare the peak shortage covered by tours with different work patterns in QL^j_k, ~~and k∈QK~~j, and find the work pattern l^* with the highest peak covered. If there are ties between two or more work patterns in QL^j_k, ~~k∈QK~~j, select the one with the highest shortage coverage per dollar

$$\frac{(\text{total shortage coverage})}{(C^j_{kl} + \Sigma_h (\text{number of hours worked on day h}) * c^j_{kni})}$$

Please replace step 37f on page 52 and continuing on to 53 with the following amended step:

37f: If tour k for agent group j is the first tour evaluated, then store the current ~~agent~~ skill group, tour schedule, and agent allocation as the best ones found so far, and go to step 37a. Otherwise, if the peak shortage covered by tour k is higher than the highest peak shortage covered by any tour and ~~agent~~ skill group considered so far, update the best ~~agent~~ skill group, tour schedule, and agent allocation with the current ones. If there is a tie between the current tour schedule and ~~agent~~ skill group, and the best ~~agent~~ skill group and tour schedule found so far, select the one with the highest

$$\frac{(\text{total shortage coverage})}{C\underline{x}^j_{ki}} \text{, for i when k} \epsilon K_j,$$

(total shortage coverage)

$$\overline{(C^j_{kl} + \Sigma_h \text{ (number of hours worked on day h)} * c^j_{kni})}$$

,for l when kεQK$_j$,

Also update the peak shortage covered, and the total shortage coverage for the current tour schedule. Go to **37g** if all ~~agent~~ <u>skill</u> groups and all tours are evaluated. Otherwise, go to **37a**.

Please replace step 37g starting on page 53 and continuing on to page 54 with the following amended step:

37g: Add the best tour schedule found in steps **37a-37f** to the schedule developed in steps 16-37. Update the values of shift, days-off, break, and agent allocation X, Y, U, W, V, <u>QX, Q, QU, QW, QV,</u> and G variables. Update shortages and excesses using the expanded schedule with the new agent.

Please replace step 39 starting on page 53 and continuing on to page 54 with the following amended step:

Step **39:** Consider the agents scheduled in the expanded schedule. Start with the most expensive ~~agent~~ skill group and consider the tours scheduled for this group. If there is a tour that can be removed without creating any shortages in any period and contact types, the current solution has redundant tours. Consider all ~~agent group~~ <u>skill groups</u> in this fashion to determine redundant agents. Go to step **40** to remove the redundant tours. Otherwise, go to step **41**.

Please replace paragraph 1 on page 54 with the following amended paragraph:

The RA algorithm can also be supplemented with a two-way interchange, or general n-way interchange step so that, when an integer feasible solution is found, the RA algorithm can make an attempt to improve it by replacing one or more scheduled tours with other tours that are not currently scheduled. This is a well-known enhancement in the field of optimization that is widely applied to a number of other optimization problems. Also, if chosen, the solution method of the invention may also be terminated prior to finding an optimal solution if, for example, (i) the objective function for the best known integer solution differs less than a pre-specified percentage from the best lower bound found by the B&C algorithm, (ii) a pre-specified number of sub-problems (e.g. one or more sub-problems) are solved by the B&C and RA algorithms, or (iii) a pre-specified amount of time elapses after locating the first integer feasible solution to the MILP. A solution that is either optimal or satisfying these conditions is referred to as a "terminal" solution. The RA algorithm may also be modified to search for an integer feasible solution to a sub-problem using any of the known heuristics (e.g. Tabu Search, Simulated Annealing, Genetic Search), or to formulate and add Gomory's cuts or other well know cuts (Wolsey, 1998) to sub-problems, change branching direction or priorities in the B&C algorithm. The optimization process outlined here doesn't exclude these extensions.

Please replace paragraph 2 on page 54 with the following amended paragraph:

Once a terminal solution is found, detailed tour schedules for agents can be developed in a number of ways. All such schedules will have the same objective value and, therefore, equally effective. First, work and non-work pattern patterns and daily shift start times are specified. Consider a tour requiring consistent daily start time times. The days-off scheduled in the solution for this tour are first assigned to individual tours. For example, if, in the terminal solution, tour type 1 has 2 agents from skill group 3 to start daily at 7:00am (that is, the variable $X^3_{1(7:00)} = 2$ in formulation (65-80)), initially these 2 agents

Please replace lines 9-11 on page 55 with the following amended lines:

in the terminal solution by the values of the decision variables are assigned to work patterns specified by the values of the work pattern variables in the terminal solution. Work pattern constraints (e.g. constraint (77)) ~~ensures~~ <u>ensure</u> that a daily shift is scheduled for